

Mathematics

Department Website: <http://math.uchicago.edu>

Program of Study

The Department of Mathematics provides an environment of research and comprehensive instruction in mathematics and applied mathematics at both undergraduate and graduate levels. Both a BA and a BS program in mathematics are offered, including a BS degree in applied mathematics and a BS degree in mathematics with a specialization in economics. Students in other fields of study may also complete a minor in mathematics; information follows the description of the major.

The requirements for a degree in mathematics or in applied mathematics express the educational intent of the Department of Mathematics; they are drawn with an eye toward the cumulative character of an education based in mathematics, the present emerging state of mathematics, and the scholarly and professional prerequisites of an academic career in mathematics.

Requirements for each bachelor's degree look to the advancement of students' general education in modern mathematics and their knowledge of its relation with the other sciences (BS) or with the other arts (BA).

Descriptions of the detailed requirements that give meaning to these educational intentions follow. Students should understand that any particular degree requirement can be modified if persuasive reasons are presented to the department; petitions to modify requirements are submitted in person to the director of undergraduate studies or to one of the departmental counselors. Students should note that only one undergraduate degree may be earned from the Department of Mathematics.

Placement

At what level does an entering student begin mathematics at the University of Chicago? Every entering student must take the mathematics placement test. This online test must be taken during the summer before arrival on campus. Scores on the mathematics placement test, combined with a student's high school record, determine the appropriate beginning mathematics course for each student:

- MATH 11200 Studies In Mathematics I
- MATH 13100 Elem Functions and Calculus I
- MATH 15100 Calculus I
- MATH 15200 Calculus II
- MATH 15300 Calculus III

Students who receive a sufficiently high score on the mathematics placement test may receive an invitation to enroll in MATH 16100 Honors Calculus I/MATH 16110 Honors Calculus I (IBL). On the basis of placement exam results, students may also be invited to sit an on-campus calculus accreditation examination prior to the start of Autumn Quarter, which would allow placement into courses at a higher level than MATH 15300 Calculus III.

Students who have scored at a high level on the online mathematics placement test will receive an invitation to take the calculus accreditation examination. Students planning to continue with higher level mathematics or other disciplines requiring advanced mathematics are urged to take this College-administered accreditation exam. On the basis of this exam, a student may receive placement into:

- MATH 15910 Introduction to Proofs in Analysis
- MATH 19520 Mathematical Methods for Social Sciences
- MATH 19620 Linear Algebra
- MATH 20000 Mathematical Methods for Physical Sciences I

Students may also be invited to begin MATH 16100 Honors Calculus I/MATH 16110 Honors Calculus I (IBL) or MATH 20700 Honors Analysis in Rn I. Students who are invited to begin Honors Calculus are encouraged to forgo credit in MATH 15100 Calculus I and/or MATH 15200 Calculus II in order to take the full Honors Calculus sequence, MATH 16100-16200-16300 Honors Calculus I-II-III or MATH 16110-16210-16310 Honors Calculus I (IBL); Honors Calculus II (IBL); Honors Calculus III (IBL)

On the basis of the Calculus Accreditation Examination or with a score of 5 on the Calculus BC Advanced Placement exam, students may also be invited to begin MATH 16100-16200-16300 Honors Calculus I-II-III or MATH 16110-16210-16310 Honors Calculus I (IBL); Honors Calculus II (IBL); Honors Calculus III (IBL). These sequences build on the sound practical background provided in strong high school calculus courses and best prepare entering students for further study in mathematics. Students who take either version of Honors Calculus forgo placement out of MATH 15100 Calculus I and/or MATH 15200 Calculus II in order to take one of these full Honors Calculus sequences.

A small number of students each year receive placement recommendations beyond Honors Calculus. Admission to MATH 20700 Honors Analysis in Rn I is by invitation only to those first-year students with superior performance on the Calculus Accreditation Examination or to those sophomores who receive a strong recommendation from their instructor

in MATH 16100-16200-16300 Honors Calculus I-II-III or MATH 16110-16210-16310 Honors Calculus I (IBL); Honors Calculus II (IBL); Honors Calculus III (IBL). Students who are granted three quarters of calculus placement on the basis of the Calculus Accreditation Examination and who do not qualify for admission to MATH 20700 Honors Analysis in Rn I will place into MATH 15910 Introduction to Proofs in Analysis. This latter option includes the possible starting points of MATH 19520 Mathematical Methods for Social Sciences, MATH 19620 Linear Algebra, or MATH 20000 Mathematical Methods for Physical Sciences I. Such students may also consult with one of the departmental counselors about the option of beginning with MATH 16100 Honors Calculus I/ MATH 16110 Honors Calculus I (IBL) so that they would be eligible for admission to Honors Analysis the following year.

Students who submit a score of 5 on the Calculus AB Advanced Placement exam in mathematics receive placement into MATH 15100 Calculus I. Students who submit scores of 4 or 5 on the AP Calculus BC exam or a 7 on the International Baccalaureate Higher Level Calculus exam receive placement into MATH 15200 Calculus II. Currently no course credit or placement is offered in the Department of Mathematics at the University of Chicago for British A-level or O-level examinations, and students with these backgrounds are strongly encouraged to take the calculus accreditation examination.

Program Requirements

Undergraduate Programs

Four bachelor's degrees are available in the Department of Mathematics: the BA in mathematics, the BS in mathematics, the BS in applied mathematics, and the BS in mathematics with specialization in economics. Programs qualifying students for the degree of BA provide more elective freedom. Programs qualifying students for the degrees of BS require more emphasis in the physical sciences, while the BS in mathematics with specialization in economics has its own set of specialized courses with more electives in economics in place of electives in the physical sciences. All degree programs, whether qualifying students for a degree in mathematics or in applied mathematics, require fulfillment of the College's general education requirements. The general education sequence in the physical sciences must be selected from either first-year chemistry or first-year physics.

Except for the BS in mathematics with specialization in economics, each degree requires at least five courses outside mathematics (detailed descriptions follow for each degree). These courses must be within the Physical Sciences Collegiate Division (PSCD) or from Computational Neuroscience (CPNS). One of these courses must complete the three-quarter sequence in basic chemistry or basic physics. At least two of these courses must be from a single department and all must be chosen from among Astronomy (20000 or above), Chemistry, Computer Science (not including 10100, 10200, 11000, 11100, or 11200), Physics (12000s or above), Geophysical Sciences, Statistics (22000 or above), CPNS, or Molecular Engineering. Graduate courses from these departments may also be used to fulfill these requirements. No courses from the Financial Mathematics program may be used in any of the undergraduate degree programs in mathematics. Please note in particular the different requirements outside of mathematics described below in the degree program for the BS in mathematics with specialization in economics.

Degree Programs in Mathematics

Students who are majoring in mathematics are required to complete: a 10000-level sequence in calculus (or to demonstrate equivalent competence on the calculus accreditation examination); either MATH 16300 Honors Calculus III or MATH 16310 Honors Calculus III (IBL) as the third quarter of the calculus sequence or MATH 15910 Introduction to Proofs in Analysis; the linear algebra course MATH 20250 Abstract Linear Algebra; a three-quarter sequence in analysis (MATH 20300-20400-20500 Analysis in Rn I-II-III or MATH 20310-20410-20510 Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated) or MATH 20700-20800-20900 Honors Analysis in Rn I-II-III); and one quarter of an algebra sequence (MATH 25400-25500 Basic Algebra I-II or MATH 25700-25800-25900 Honors Basic Algebra I-II-III). Students may not use both MATH 15910 Introduction to Proofs in Analysis and (MATH 16300 Honors Calculus III/MATH 16310 Honors Calculus III (IBL)) to meet major or minor requirements. For students whose first mathematics course at the University of Chicago is MATH 20700 Honors Analysis in Rn I, the MATH 15910 Introduction to Proofs in Analysis/MATH 16300 Honors Calculus III/MATH 16310 Honors Calculus III (IBL) requirement is waived. For students who complete MATH 20700 Honors Analysis in Rn I, the MATH 20250 Abstract Linear Algebra requirement is waived, but the student must then take an additional course from the List of Approved Courses.

Candidates for the BA and BS in mathematics take at least one course in basic algebra. BA candidates may opt for the first quarter of either the regular or the honors sequence (or MATH 25700-25800-25900 Honors Basic Algebra I-II-III), whereas candidates for the BS degree must take the first two quarters of one of the two sequences. MATH 25700-25800-25900 Honors Basic Algebra I-II-III is designated as an honors version of Basic Algebra. Registration for this course is the option of the individual student, but consultation with one of the departmental counselors is strongly advised.

The remaining mathematics courses needed in the programs (three for the BA, two for the BS) must be selected, with due regard for prerequisites, from the following list of approved mathematics courses. Note that STAT 25100 Introduction to Mathematical Probability or STAT 25150 Introduction to Mathematical Probability-A also meet the requirement. BA candidates may include MATH 25500 Basic Algebra II or MATH 25800 Honors Basic Algebra II. Mathematics courses in the Paris Mathematics program each Spring Quarter may also be used to meet this requirement.

List of Approved Courses

MATH 17500	Basic Number Theory	100
MATH 17600	Basic Geometry	100
MATH 21100	Basic Numerical Analysis	100

MATH 21200	Advanced Numerical Analysis	100
MATH 23500	Markov Chains, Martingales, and Brownian Motion	100
MATH 24100	Topics in Geometry	100
MATH 24200	Algebraic Number Theory	100
MATH 24300	Intro To Algebraic Curves	100
MATH 24400	Introduction to Algebraic Geometry	100
MATH 25900	Honors Basic Algebra III	100
MATH 26200	Point-Set Topology	100
MATH 26300	Introduction to Algebraic Topology	100
MATH 26700	Introduction to Representation Theory of Finite Groups	100
MATH 26800	Introduction to Commutative Algebra	100
MATH 27000	Basic Complex Variables	100
MATH 27100	Measure and Integration	100
MATH 27200	Basic Functional Analysis	100
MATH 27300	Basic Theory of Ordinary Differential Equations	100
MATH 27400	Introduction to Differentiable Manifolds and Integration on Manifolds	100
MATH 27500	Basic Theory of Partial Differential Equations	100
MATH 27700	Mathematical Logic-I	100
MATH 27800	Mathematical Logic II	100
MATH 28000	Introduction to Formal Languages	100
MATH 28100	Introduction to Complexity Theory	100
MATH 28410	Honors Combinatorics	100
MATH 29200	Chaos, Complexity And Computers	100
MATH 29700	Proseminar in Mathematics *	100
MATH 30200	Computability Theory I	100
MATH 30300	Computability Theory II	100
MATH 30900	Model Theory I	100
MATH 31000	Model Theory II	100
MATH 31200	Analysis I	100
MATH 31300	Analysis II	100
MATH 31400	Analysis III	100
MATH 31700	Topology and Geometry I	100
MATH 31800	Topology/Geometry-2	100
MATH 31900	Topology/Geometry - 3	100
MATH 32500	Algebra I	100
MATH 32600	Algebra II	100
MATH 32700	Algebra III	100
STAT 25100	Introduction to Mathematical Probability	100
STAT 25150	Introduction to Mathematical Probability-A	100

* as approved

BS candidates are further required to select a minor field, which consists of three additional courses that are outside the Department of Mathematics and either are within the same department in the Physical Sciences Collegiate Division (PSCD) or are among Computational Neuroscience (CPNS) courses. These courses must be chosen in consultation with one of the departmental counselors.

Summaries of Requirements

Summary of Requirements: Mathematics BA

GENERAL EDUCATION

One of the following sequences:		200
CHEM 10100 & CHEM 10200	Introductory General Chemistry I and Introductory General Chemistry II	
CHEM 11100-11200	Comprehensive General Chemistry I-II (or equivalent) *	
PHYS 12100-12200	General Physics I-II (or higher) * +	
One of the following sequences:		200

MATH 13100-13200	Elementary Functions and Calculus I-II	
MATH 15100-15200	Calculus I-II	
MATH 16100-16200	Honors Calculus I-II *	
MATH 16110 & MATH 16210	Honors Calculus I (IBL) and Honors Calculus II (IBL) *	
Total Units		400
MAJOR		
One of the following:		100
CHEM 11300	Comprehensive General Chemistry III (or equivalent) *	
PHYS 12300	General Physics III (or higher) * +	
One of the following: **		100
MATH 16300	Honors Calculus III	
MATH 16310	Honors Calculus III (IBL)	
MATH 15910	Introduction to Proofs in Analysis	
MATH 20250	Abstract Linear Algebra	100
One of the following:		300
MATH 20300-20400-20500	Analysis in R^n I-II-III	
MATH 20310-20410-20510	Analysis in R^n I (accelerated); Analysis in R^n II (accelerated); Analysis in R^n III (accelerated)	
MATH 20700-20800-20900	Honors Analysis in R^n I-II-III ^	
Two mathematics courses chosen from the List of Approved Courses		200
Four courses within the PSCD or from CPNS but outside of mathematics, at least two of which should be taken in a single department ***		400
BA Specific		
One of the following:		100
MATH 25400	Basic Algebra-I	
MATH 25700	Honors Basic Algebra I	
One of the following:		100
MATH 25500	Basic Algebra II	
MATH 25800	Honors Basic Algebra II	
A course from the List of Approved Courses		
Total Units		1400
Summary of Requirements: Mathematics BS		
GENERAL EDUCATION		
One of the following sequences:		200
CHEM 10100 & CHEM 10200	Introductory General Chemistry I and Introductory General Chemistry II	
CHEM 11100-11200	Comprehensive General Chemistry I-II (or equivalent) *	
PHYS 12100-12200	General Physics I-II (or higher) * +	
One of the following sequences:		200
MATH 13100-13200	Elementary Functions and Calculus I-II	
MATH 15100-15200	Calculus I-II	
MATH 16100-16200	Honors Calculus I-II *	
MATH 16110 & MATH 16210	Honors Calculus I (IBL) and Honors Calculus II (IBL)	
Total Units		400
MAJOR		
One of the following:		100
CHEM 11300	Comprehensive General Chemistry III (or equivalent) *	
PHYS 12300	General Physics III (or higher) * +	
One of the following: **		100

MATH 16300	Honors Calculus III	
MATH 16310	Honors Calculus III (IBL)	
MATH 15910	Introduction to Proofs in Analysis	
MATH 20250	Abstract Linear Algebra	100
One of the following:		300
MATH 20300-20400-20500	Analysis in Rn I-II-III	
MATH 20310-20410-20510	Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated)	
MATH 20700-20800-20900	Honors Analysis in Rn I-II-III ^	
Two Mathematics courses chosen from the List of Approved Courses		200
Four courses within the PSCD or from CPNS but outside of mathematics, at least two of which should be taken in a single department ***		400
BS Specific		
One of the following:		200
MATH 25400-25500	Basic Algebra I-II	
MATH 25700 & MATH 25800	Honors Basic Algebra I and Honors Basic Algebra II	
Three courses that are not MATH courses but are either from the same PSCD department or CPNS		300
Total Units		1700

* Credit may be granted by examination.

** Students who complete (or receive credit for) MATH 13300 Elementary Functions and Calculus III or MATH 15300 Calculus III must use these courses as general electives, and MATH 15910 Introduction to Proofs in Analysis must be completed for the major.

*** May include BIOS 24231 Methods in Computational Neuroscience and BIOS 24232 Computational Approaches to Cognitive Neuroscience, or AP credit for STAT 22000 Statistical Methods and Applications, CHEM 11100 Comprehensive General Chemistry I, and/or PHYS 12100-12200 General Physics I-II. May *not* include CMSC 10100 Introduction to Programming for the World Wide Web I, CMSC 10200 Introduction to Programming for the World Wide Web II, CMSC 11000 Multimed Prog: Interdisc Art-1, CMSC 11100 Multimedia Programming as an Interdisciplinary Art II, or any PHSC course.

+ The sequence PHYS 13100-13200 Mechanics; Electricity and Magnetism is recommended for mathematics majors.

^ Students who complete MATH 20700 Honors Analysis in Rn I will not be required to take MATH 20250 Abstract Linear Algebra; in its place they will take an additional course from the List of Approved Courses.

Degree Program in Applied Mathematics

Candidates for the BS in applied mathematics all take prescribed courses in numerical analysis, algebra, complex variables, ordinary differential equations, and partial differential equations. In addition, candidates are required to select, in consultation with one of the departmental counselors, a secondary field, which consists of three additional courses from a single department that is outside the Department of Mathematics but within the Physical Sciences Collegiate Division or among Computational Neuroscience (CPNS) courses.

Summary of Requirements: BS in Applied Mathematics

GENERAL EDUCATION

One of the following:		200
CHEM 10100 & CHEM 10200	Introductory General Chemistry I and Introductory General Chemistry II	
CHEM 11100-11200	Comprehensive General Chemistry I-II (or equivalent) *	
PHYS 12100-12200	General Physics I-II (or higher) * +	
One of the following:		200
MATH 13100-13200	Elementary Functions and Calculus I-II	
MATH 15100-15200	Calculus I-II	
MATH 16100-16200	Honors Calculus I-II *	
MATH 16110 & MATH 16210	Honors Calculus I (IBL) and Honors Calculus II (IBL)	
Total Units		400

MAJOR

One of the following:		100
CHEM 11300	Comprehensive General Chemistry III (or equivalent) *	
PHYS 12300	General Physics III (or higher) * +	
One of the following: **		100
MATH 16300	Honors Calculus III	
MATH 16310	Honors Calculus III (IBL)	
MATH 15910	Introduction to Proofs in Analysis	
MATH 20250	Abstract Linear Algebra	100
One of the following:		300
MATH 20300-20400-20500	Analysis in Rn I-II-III	
MATH 20310-20410-20510	Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated)	
MATH 20700-20800-20900	Honors Analysis in Rn I-II-III	
One of the following:		100
MATH 21100	Basic Numerical Analysis	
MATH 21200	Advanced Numerical Analysis	
One of the following:		100
MATH 25400	Basic Algebra-I	
MATH 25700	Honors Basic Algebra I	
All three of the following courses:		300
MATH 27000	Basic Complex Variables	
MATH 27300	Basic Theory of Ordinary Differential Equations	
MATH 27500	Basic Theory of Partial Differential Equations	
Six courses that are not MATH courses but are either within the PSCD or from CPNS, at least three of which should be taken in a single department **		600
Total Units		1700

* Credit may be granted by examination.

** See restrictions on certain courses listed under previous summary.

+ The sequence PHYS 13100-13200 Mechanics; Electricity and Magnetism is recommended for mathematics majors.

Degree Program in Mathematics with Specialization in Economics

This program is a version of the BS in mathematics. The BS degree is in mathematics with the designation "with specialization in economics" included on the final transcript. Candidates are required to complete a yearlong sequence in calculus, MATH 15910 Introduction to Proofs in Analysis if the calculus sequence did not terminate with MATH 16300 Honors Calculus III/MATH 16310 Honors Calculus III (IBL), the one-quarter course MATH 20250 Abstract Linear Algebra, a yearlong sequence in analysis (MATH 20300-20400-20500 Analysis in Rn I-II-III or MATH 20310-20410-20510 Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated) or MATH 20700-20800-20900 Honors Analysis in Rn I-II-III), and one quarter of abstract algebra (MATH 25400 Basic Algebra-I or MATH 25700 Honors Basic Algebra I), and earn a grade of at least C- in each course. Students must also take STAT 25100 Introduction to Mathematical Probability or STAT 25150 Introduction to Mathematical Probability-A. The remaining two mathematics courses must be among the following five: MATH 27000 Basic Complex Variables, MATH 27100 Measure and Integration, MATH 27200 Basic Functional Analysis, MATH 27300 Basic Theory of Ordinary Differential Equations, or MATH 23500 Markov Chains, Martingales, and Brownian Motion. A C average or higher must be earned in these two courses.

In addition to the third quarter of basic chemistry or basic physics, the eight courses required outside the Department of Mathematics must include STAT 23400 Statistical Models and Methods or STAT 24400 Statistical Theory and Methods I. The remaining seven courses should be in the Department of Economics and must include ECON 20000-20100-20200 The Elements of Economic Analysis I-II-III or ECON 20010-ECON 20110-ECON 20210 The Elements of Economic Analysis: Honors I-II-III and either ECON 20900 Econometrics: Honors or ECON 21000 Econometrics A. The remaining two courses may be chosen from any undergraduate economics course numbered higher than ECON 20210 The Elements of Economic Analysis III Honors. A University of Chicago Booth School of Business course may be considered for elective credit if the course requires the equivalent of ECON 20100 as a prerequisite and is numbered as a Chicago Booth 40000 or higher course. Additionally, the course needs to pertain to the application of economic theory to a course subject that is not offered by the Department of Economics. Courses such as accounting, investments, and entrepreneurship will not be considered for economics elective credit. Consideration for elective credit must be done by petition before a student registers for the

course. There will be no retroactive consideration for credit. Students must earn a grade of C or higher in each course taken in economics to be eligible for this degree.

It is recommended that students considering graduate work in economics use some of their electives to include at least one programming course (CMSC 15100 Intro To Computer Science-I is strongly recommended) and an additional course in statistics (STAT 24400-24500 Statistical Theory and Methods I-II or STAT 24410 Statistical Theory and Methods Ia and STAT 24500 Statistical Theory and Methods II are appropriate two-quarter sequences). Students planning to apply to graduate economics programs are strongly encouraged to meet with one of the economics undergraduate program directors before the beginning of their third year.

Summary of Requirements: BS in Mathematics with Specialization in Economics

GENERAL EDUCATION

One of the following sequences:	200
CHEM 10100 & CHEM 10200	Introductory General Chemistry I and Introductory General Chemistry II
CHEM 11100-11200	Comprehensive General Chemistry I-II (or equivalent) *
PHYS 12100-12200	General Physics I-II (or higher) * +
One of the following sequences:	200
MATH 13100-13200	Elementary Functions and Calculus I-II
MATH 15100-15200	Calculus I-II
MATH 16100-16200	Honors Calculus I-II *
MATH 16110 & MATH 16210	Honors Calculus I (IBL) and Honors Calculus II (IBL)
Total Units	400

MAJOR

One of the following:	100
CHEM 11300	Comprehensive General Chemistry III (or higher) *
PHYS 12300	General Physics III (or higher) * +
One of the following: **	100
MATH 16300	Honors Calculus III
MATH 16310	Honors Calculus III (IBL)
MATH 15910	Introduction to Proofs in Analysis
MATH 20250	Abstract Linear Algebra
One of the following:	300
MATH 20300-20400-20500	Analysis in R^n I-II-III
MATH 20310-20410-20510	Analysis in R^n I (accelerated); Analysis in R^n II (accelerated); Analysis in R^n III (accelerated)
MATH 20700-20800-20900	Honors Analysis in R^n I-II-III
One of the following:	100
MATH 25400	Basic Algebra-I
MATH 25700	Honors Basic Algebra I
Two of the following:	200
MATH 27000	Basic Complex Variables
MATH 27100	Measure and Integration
MATH 27200	Basic Functional Analysis
MATH 27300	Basic Theory of Ordinary Differential Equations
MATH 23500	Markov Chains, Martingales, and Brownian Motion
One of the following:	100
STAT 25100	Introduction to Mathematical Probability
STAT 25150	Introduction to Mathematical Probability-A
One of the following:	100
STAT 23400	Statistical Models and Methods
STAT 24400	Statistical Theory and Methods I
STAT 24410	Statistical Theory and Methods Ia

One of the following:		300
ECON 20000-20100-20200	The Elements of Economic Analysis I-II-III	
ECON 20010-20110-20210	The Elements of Economic Analysis: Honors I-II-III	
One of the following:		100
ECON 21020	Econometrics	
ECON 21030	Econometrics - Honors	
Three Economics courses numbered higher than 20210		300
Total Units		1800

* Credit may be granted by examination.

** See restrictions on certain courses listed under earlier summary.

+ The sequence PHYS 13100-13200 Mechanics; Electricity and Magnetism is recommended for mathematics majors.

Grading

Subject to College grading requirements and grading requirements for the major and with consent of instructor, students (except students who are majoring in mathematics or applied mathematics) may take any mathematics course beyond the second quarter of calculus for either a quality grade or for P/F grading. A Pass grade is given only for work of C- quality or higher.

All courses taken to meet requirements in the mathematics major must be taken for quality grades. A grade of C- or higher must be earned in each calculus, analysis, or algebra course; and an overall grade average of C or higher must be earned in the remaining mathematics courses that a student uses to meet requirements for the major. Students must earn a grade of C or higher in each course taken in economics for the degree in mathematics with a specialization in economics. Mathematics or applied mathematics students may take any 20000-level mathematics courses elected beyond program requirements for P/F grading.

Incompletes are given in the Department of Mathematics only to those students who have done some work of passing quality and who are unable to complete all the course work by the end of the quarter. Arrangements are made between the instructor and the student.

Honors

The BA or BS with honors is awarded to students who, while meeting requirements for one of the mathematics degrees, also meet the following requirements: (1) a GPA of 3.25 or higher in mathematics courses and a 3.0 or higher overall; (2) no grade below C- and no grade of W in any mathematics course; (3) completion of at least one honors sequence (either MATH 20700-20800-20900 Honors Analysis in Rn I-II-III or MATH 25700-25800-25900 Honors Basic Algebra I-II-III) with grades of B- or higher in each quarter; and (4) completion with a grade of B- or higher of at least five mathematics courses chosen from the list that follows so that at least one course comes from each group (i.e., algebra, analysis, and topology). No course may be used to satisfy both requirement (3) and requirement (4). If both honors sequences are taken, one sequence may be used for requirement (3) and one sequence may be used for up to three of the five courses in requirement (4).

Algebra Courses

MATH 24100	Topics in Geometry	100
MATH 24200	Algebraic Number Theory	100
MATH 24300	Intro To Algebraic Curves	100
MATH 24400	Introduction to Algebraic Geometry	100
MATH 25700	Honors Basic Algebra I	100
MATH 25800	Honors Basic Algebra II	100
MATH 25900	Honors Basic Algebra III	100
MATH 26700	Introduction to Representation Theory of Finite Groups	100
MATH 26800	Introduction to Commutative Algebra	100
MATH 27700	Mathematical Logic-I	100
MATH 27800	Mathematical Logic II	100
MATH 28410	Honors Combinatorics	100
MATH 32500	Algebra I	100
MATH 32600	Algebra II	100
MATH 32700	Algebra III	100

Analysis Courses

MATH 20700	Honors Analysis in Rn I	100
MATH 20800	Honors Analysis in Rn II	100
MATH 20900	Honors Analysis in Rn III	100
MATH 23500	Markov Chains, Martingales, and Brownian Motion	100
MATH 27000	Basic Complex Variables	100
MATH 27100	Measure and Integration	100
MATH 27200	Basic Functional Analysis	100
MATH 27300	Basic Theory of Ordinary Differential Equations	100
MATH 27400	Introduction to Differentiable Manifolds and Integration on Manifolds	100
MATH 27500	Basic Theory of Partial Differential Equations	100
MATH 31200	Analysis I	100
MATH 31300	Analysis II	100
MATH 31400	Analysis III	100

Topology Courses

MATH 26200	Point-Set Topology	100
MATH 26300	Introduction to Algebraic Topology	100
MATH 31700	Topology and Geometry I	100
MATH 31800	Topology/Geometry-2	100
MATH 31900	Topology/Geometry - 3	100

With departmental approval, MATH 29700 Proseminar in Mathematics, or any course(s) in the Paris Mathematics Program, may be chosen so that it falls in one of the three groups. One of the three Paris courses each year will be designated as a replacement for MATH 25500 Basic Algebra II for students wishing to complete the BS degree. Additionally, one of the three Paris courses each year will be designated as a replacement for MATH 25900 Honors Basic Algebra III for candidates who are working toward graduation with honors. Courses taken for the honors requirements (3) and (4) also may be counted toward courses taken to meet requirements for the major. Students who wish to be considered for honors should consult with one of the departmental counselors no later than Spring Quarter of their third year.

Minor Program in Mathematics

The minor in mathematics requires a total of six or seven courses in mathematics, depending on whether or not MATH 15910 Introduction to Proofs in Analysis, MATH 16300 Honors Calculus III or MATH 16310 Honors Calculus III (IBL) is required in another degree program. If it is not used elsewhere, MATH 15910 Introduction to Proofs in Analysis, MATH 16300 Honors Calculus III or MATH 16310 Honors Calculus III (IBL) must be included in the minor, for a total of seven courses. The remaining six courses must include the linear algebra course MATH 20250 Abstract Linear Algebra, a three-course sequence in analysis MATH 20300-20400-20500 Analysis in Rn I-II-III or MATH 20310-20410-20510 Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated) or MATH 20700-20800-20900 Honors Analysis in Rn I-II-III), and the first course in one of the algebra sequences (MATH 25400 Basic Algebra-I or MATH 25700 Honors Basic Algebra I). The sixth course may be chosen from either the second course in one of the algebra sequences (MATH 25500 Basic Algebra II or MATH 25800 Honors Basic Algebra II) or a mathematics course numbered 23000 or higher chosen in consultation with the director of undergraduate studies or one of the departmental counselors. A student who completes MATH 20700 Honors Analysis in Rn I is not obligated to take MATH 20250 Abstract Linear Algebra, but should instead select another mathematics course numbered 23000 or higher. Under special circumstances and to avoid double counting, students may also use mathematics courses numbered 23000 or higher to substitute for up to two quarters of analysis or algebra, if these are required in another degree program.

No course in the minor can be double counted with the student's major(s) or with other minors; nor can it be counted toward general education requirements. Students must earn a grade of at least C- in each of the courses in the mathematics minor. More than one-half of the requirements for a minor must be met by registering for courses bearing University of Chicago course numbers.

Students must meet with the director of undergraduate studies or one of the departmental counselors by Spring Quarter of their third year to declare their intention to complete a minor program in mathematics and to obtain approval for the minor on a form obtained from their College adviser. Courses for the minor are chosen in consultation with the director of undergraduate studies or one of the departmental counselors.

Paris Mathematics Program (<http://study-abroad.uchicago.edu/programs/paris-mathematics>)

Each Spring Quarter, the Department of Mathematics offers a study abroad opportunity for students to take upper-level mathematics electives at the University's Center in Paris. Departmental faculty offer three successive three-week courses in specialized topics, and students also take a French language course from local French faculty. Students should have completed one of the analysis sequences (MATH 20300-20400-20500 Analysis in Rn I-II-III or MATH 20310-20410-20510 Analysis in Rn I (accelerated); Analysis in Rn II (accelerated); Analysis in Rn III (accelerated) or MATH 20700-20800-20900 Honors Analysis in Rn I-II-III) and at least one quarter of one of the algebra sequences

(MATH 25400 Basic Algebra-I or MATH 25700 Honors Basic Algebra I) before attending the Paris program. First round applications are due the prior Spring Quarter and should be submitted to the Study Abroad office. If the program does not reach maximum capacity, second round applications will also be accepted in the Autumn Quarter.

Joint Degree Programs

BA/MS or BS/MS in Mathematics

Qualified College students may receive both a bachelor's and a master's degree in mathematics concurrently at the end of their studies in the College. Qualification consists of satisfying all requirements of each degree in mathematics. To be eligible for the joint program, a student must begin MATH 20700 Honors Analysis in Rn I in the Autumn Quarter of the student's first year. By following a program of prescribed undergraduate course sequences in mathematics and succeeding in all courses with grades no lower than A-, the student becomes eligible to enroll in graduate courses in mathematics in the student's third year. While only a few students complete the joint BA/MS program, many undergraduates enroll in graduate-level mathematics courses. Admission to all mathematics graduate courses requires prior written consent of the director or co-director of undergraduate studies.

Students should submit their application for the joint program to one of the departmental counselors as soon as possible, but no later than the Winter Quarter of their third year.

Mathematics Courses

MATH 11200-11300. Studies in Mathematics I-II.

MATH 11200 AND 11300 cover the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. MATH 11200 addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. MATH 11300's main topic is symmetry and geometry, including a study of polygons, Euclidean construction, polyhedra, group theory, and topology. These courses emphasize the understanding of ideas and the ability to express them through rigorous mathematical arguments. While students may take MATH 11300 without having taken MATH 11200, it is recommended that MATH 11200 be taken first. Either course in this sequence meets the general education requirement in mathematical sciences. These courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence.

MATH 11200. Studies In Mathematics I. 100 Units.

MATH 11200 AND 11300 cover the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. MATH 11200 addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. These courses emphasize the understanding of ideas and the ability to express them through rigorous mathematical arguments. While students may take MATH 11300 without having taken MATH 11200, it is recommended that MATH 11200 be taken first. Either course in this sequence meets the general education requirement in mathematical sciences. These courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence.

Terms Offered: Autumn, Spring

MATH 11300. Studies In Mathematics-2. 100 Units.

MATH 11200 AND 11300 cover the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. MATH 11200 addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. These courses emphasize the understanding of ideas and the ability to express them through rigorous mathematical arguments. While students may take MATH 11300 without having taken MATH 11200, it is recommended that MATH 11200 be taken first. Either course in this sequence meets the general education requirement in mathematical sciences. These courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence.

Terms Offered: Winter

Prerequisite(s): MATH 11200 recommended

MATH 11300. Studies In Mathematics-2. 100 Units.

MATH 11200 AND 11300 cover the basic conceptual foundations of mathematics by examining the ideas of number and symmetry. MATH 11200 addresses number theory, including a study of the rules of arithmetic, integral domains, primes and divisibility, congruences, and modular arithmetic. These courses emphasize the understanding of ideas and the ability to express them through rigorous mathematical arguments. While students may take MATH 11300 without having taken MATH 11200, it is recommended that MATH 11200 be taken first. Either course in this sequence meets the general education requirement in mathematical sciences. These courses are at the level of difficulty of the MATH 13100-13200-13300 calculus sequence.

Terms Offered: Winter

Prerequisite(s): MATH 11200 recommended

MATH 13100-13200-13300. Elementary Functions and Calculus I-II-III.

MATH 13100-13200-13300 is a sequence in calculus for students who need some precalculus reinforcement. The sequence completes the necessary background and covers basic calculus in three quarters. This is achieved through three regular one-hour class meetings and two mandatory one-and-one-half-hour tutorial sessions each week. A class is divided into tutorial groups of about eight students each, and these meet with an undergraduate junior tutor for problem solving related to the course. Students completing MATH 13100-13200-13300 have a command of calculus equivalent to that obtained in MATH 15100-15200-15300. Students may not take the first two quarters of this sequence for P/F grading. MATH 13100-13200 meets the general education requirement in the mathematical sciences.

MATH 13100. Elem Functions and Calculus I. 100 Units.

MATH 13100 gives a careful treatment of limits, the continuity and differentiability of algebraic functions, and applications of the derivative.

Terms Offered: Autumn Winter

Prerequisite(s): Invitation only, based on adequate performance on the mathematics placement test

MATH 13200. Elem Functions and Calculus II. 100 Units.

Topics examined in MATH 13200 include applications of differentiation; exponential, logarithmic, and trigonometric functions; the definite integral and the Fundamental Theorem of Calculus, and applications of the integral.

Terms Offered: Spring Winter

Prerequisite(s): MATH 13100

MATH 13300. Elementary Functions and Calculus III. 100 Units.

In MATH 13300, subjects include more applications of the definite integral, an introduction to infinite sequences and series and Taylor expansions. MATH 13300 also includes an introduction to multivariable calculus, such as functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables.

Terms Offered: Spring

Prerequisite(s): MATH 13200

MATH 13200. Elem Functions and Calculus II. 100 Units.

Topics examined in MATH 13200 include applications of differentiation; exponential, logarithmic, and trigonometric functions; the definite integral and the Fundamental Theorem of Calculus, and applications of the integral.

Terms Offered: Spring Winter

Prerequisite(s): MATH 13100

MATH 13300. Elementary Functions and Calculus III. 100 Units.

In MATH 13300, subjects include more applications of the definite integral, an introduction to infinite sequences and series and Taylor expansions. MATH 13300 also includes an introduction to multivariable calculus, such as functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables.

Terms Offered: Spring

Prerequisite(s): MATH 13200

MATH 15100-15200-15300. Calculus I-II-III.

This is the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. All Autumn Quarter offerings of MATH 15100, 15200, and 15300 begin with a rigorous treatment of limits and limit proofs. Students may not take the first two quarters of this sequence for P/F grading. MATH 15100-15200 meets the general education requirement in mathematical sciences.

MATH 15100. Calculus I. 100 Units.

This is the first course in the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. MATH 15100 undertakes a careful treatment of limits, the differentiation of algebraic and transcendental functions, applications of differentiation, and the Mean Value Theorem. All Autumn Quarter offerings of MATH 15100 begin with a rigorous treatment of limits and limit proofs. Students may not take the first two quarters of this sequence for P/F grading. MATH 15100-15200 meets the general education requirement in mathematical sciences.

Terms Offered: Autumn

Prerequisite(s): Superior performance on the mathematics placement test, or MATH 10500

MATH 15200. Calculus II. 100 Units.

This is the second course in the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. MATH 15200 covers integration, techniques of integration, applications of the integral, and transcendental functions. All Autumn Quarter offerings of MATH 15200 begin with a rigorous treatment of limits and limit proofs. Students may not take the first two quarters of this sequence for P/F grading. MATH 15100-15200 meets the general education requirement in mathematical sciences.

Terms Offered: Autumn Winter

Prerequisite(s): MATH 15100; or placement based on the Calculus Accreditation Exam or appropriate AP score

MATH 15300. Calculus III. 100 Units.

This is the third course in the regular calculus sequence in the department. MATH 15300 covers applications of integration, an introduction to infinite sequences and series and Taylor expansions, and an introduction to multivariable calculus including functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables. All Autumn Quarter offerings of MATH 15300 begin with a rigorous treatment of limits and limit proofs

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 15200; or placement based on the Calculus Accreditation Exam or appropriate AP score

MATH 15200. Calculus II. 100 Units.

This is the second course in the regular calculus sequence in the department. Students entering this sequence are to have mastered appropriate precalculus material and, in many cases, have had some previous experience with calculus in high school or elsewhere. MATH 15200 covers integration, techniques of integration, applications of the integral, and transcendental functions. All Autumn Quarter offerings of MATH 15200 begin with a rigorous treatment of limits and limit proofs. Students may not take the first two quarters of this sequence for P/F grading. MATH 15100-15200 meets the general education requirement in mathematical sciences.

Terms Offered: Autumn Winter

Prerequisite(s): MATH 15100; or placement based on the Calculus Accreditation Exam or appropriate AP score

MATH 15300. Calculus III. 100 Units.

This is the third course in the regular calculus sequence in the department. MATH 15300 covers applications of integration, an introduction to infinite sequences and series and Taylor expansions, and an introduction to multivariable calculus including functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables. All Autumn Quarter offerings of MATH 15300 begin with a rigorous treatment of limits and limit proofs

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 15200; or placement based on the Calculus Accreditation Exam or appropriate AP score

MATH 15910. Introduction to Proofs in Analysis. 100 Units.

This course is intended for students who are making the transition from MATH 13300 or 15300 to MATH 20250 and MATH 20300, or for students who need more preparation in learning to read and write proofs. This course covers the fundamentals of theoretical mathematics and prepares students for upper-level mathematics courses beginning with MATH 20250 and MATH 20300. Topics include the axioms for the real numbers, completeness and the least upper bound property, the topology of the real line, and sequences and series of real and complex numbers. Students who are majoring or minoring in mathematics may not use both MATH 15910 and MATH 16300 to meet program requirements.

Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 15300 or MATH 13300 or superior performance on the Calculus Accreditation Exam

MATH 16100-16200-16300. Honors Calculus I-II-III.

MATH 16100-16200-16300 is an honors version of MATH 15100-15200-15300. A student with a strong background in the problem-solving aspects of one-variable calculus may, by suitable achievement on the Calculus Accreditation Exam, be invited to register for MATH 16100-16200-16300. This sequence emphasizes the theoretical aspects of one-variable analysis and, in particular, the consequences of completeness in the real number system. MATH 16300 also includes an introduction to multivariable calculus. At least one section of this sequence is offered as an inquiry-based learning (IBL) course. Students interested in IBL should have fluency in spoken English and an AP score of 5 on the BC Calculus exam or placement into MATH 15300. Students may not take the first two quarters of this sequence for P/F grading. MATH 16100-16200 meets the general education requirement in mathematical sciences.

MATH 16100. Honors Calculus I. 100 Units.

MATH 16100-16200-16300 is an honors version of MATH 15100-15200-15300. A student with a strong background in the problem-solving aspects of one-variable calculus may, by suitable achievement on the Calculus Accreditation Exam, be invited to register for MATH 16100-16200-16300. This sequence emphasizes the theoretical aspects of one-variable analysis and, in particular, the consequences of completeness in the real number system. MATH 16300 also includes an introduction to multivariable calculus. At least one section of this sequence is offered as an inquiry-based learning (IBL) course. Students interested in IBL should have fluency in spoken English and an AP score of 5 on the BC Calculus exam or placement into MATH 15300. Students may not take the first two quarters of this sequence for P/F grading. MATH 16100-16200 meets the general education requirement in mathematical sciences. MATH 16100 emphasizes the theoretical aspects of one-variable analysis and, in particular, the consequences of completeness in the real number system. Topics include a rigorous treatment of the real numbers and the least upper bound property, limits, continuity, uniform continuity, and differentiation. Prerequisite(s): Invitation only based on superior performance on the Calculus Accreditation Examination

Terms Offered: Autumn

Prerequisite(s): Invitation only based on superior performance on the Calculus Accreditation Examination

MATH 16200. Honors Calculus II. 100 Units.

MATH 16200 covers integration, the Fundamental Theorem of Calculus, transcendental functions, and other topics.

Terms Offered: Winter

Prerequisite(s): MATH 16100

MATH 16300. Honors Calculus III. 100 Units.

MATH 16300 covers sequences and series, power series, and Taylor series. It also includes an introduction to multivariable calculus, such as functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables.

Terms Offered: Spring

Prerequisite(s): MATH 16200

MATH 16200. Honors Calculus II. 100 Units.

MATH 16200 covers integration, the Fundamental Theorem of Calculus, transcendental functions, and other topics.

Terms Offered: Winter

Prerequisite(s): MATH 16100

MATH 16300. Honors Calculus III. 100 Units.

MATH 16300 covers sequences and series, power series, and Taylor series. It also includes an introduction to multivariable calculus, such as functions of several real variables, partial derivatives, gradients, and the total derivative, and integration of functions of several variables.

Terms Offered: Spring

Prerequisite(s): MATH 16200

MATH 16110-16210-16310. Honors Calculus I (IBL); Honors Calculus II (IBL); Honors Calculus III (IBL)

This sequence is an Inquiry Based Learning version of MATH 16100-16200-16300 Honors Calculus I-II-III. In this alternate version of Honors Calculus, rather than having lectures from instructors, students are given "scripts" of carefully ordered theorems whose proofs they prepare outside of class and then present in class for comment and discussion. MATH 16110-16210 meets the general education requirement in mathematical sciences.

MATH 16110. Honors Calculus I (IBL) 100 Units.

MATH 16110 gives a rigorous axiomatic treatment of the continuum and its topological properties.

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): Invitation only based on superior performance on the Calculus Accreditation Examination

MATH 16210. Honors Calculus II (IBL) 100 Units.

MATH 16210 puts an arithmetic structure on the continuum, and constructs the real numbers via Dedekind cuts. There follows a rigorous treatment of limits, continuity, differentiability, integrability, and the Fundamental Theorem of Calculus.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 16110

MATH 16310. Honors Calculus III (IBL) 100 Units.

MATH 16310 continues the rigorous treatment of single-variable Calculus with a discussion of infinite series. There follows an introduction to the main ideas of multivariable Calculus, including functions of several real variables, partial derivatives, gradients, the total derivative, and integration of functions of several variables.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 16210

MATH 16210. Honors Calculus II (IBL) 100 Units.

MATH 16210 puts an arithmetic structure on the continuum, and constructs the real numbers via Dedekind cuts. There follows a rigorous treatment of limits, continuity, differentiability, integrability, and the Fundamental Theorem of Calculus.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 16110

MATH 16310. Honors Calculus III (IBL) 100 Units.

MATH 16310 continues the rigorous treatment of single-variable Calculus with a discussion of infinite series. There follows an introduction to the main ideas of multivariable Calculus, including functions of several real variables, partial derivatives, gradients, the total derivative, and integration of functions of several variables.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 16210

MATH 17500. Basic Number Theory. 100 Units.

This course covers basic properties of the integers following from the division algorithm, primes and their distribution, and congruences. Additional topics include existence of primitive roots, arithmetic functions, quadratic reciprocity, and transcendental numbers. The subject is developed in a leisurely fashion, with many explicit examples.

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): MATH 16300 or MATH 16310 or MATH 15910 or MATH 15900 or MATH 19900

MATH 17600. Basic Geometry. 100 Units.

This course covers advanced topics in geometry, including Euclidean geometry, spherical geometry, and hyperbolic geometry. We emphasize rigorous development from axiomatic systems, including the approach of Hilbert. Additional topics include lattice point geometry, projective geometry, and symmetry.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 16300 or MATH 16310 or MATH 15910 or MATH 15900 or MATH 19900

MATH 19520. Mathematical Methods for Social Sciences. 100 Units.

MATH 19520 is a course in mathematical techniques for students in the social sciences. It covers the basic topics of multivariable calculus including vectors and vector functions, partial derivatives, multiple integrals, and Lagrange multipliers. It also covers an introduction to optimization, including linear programming, the simplex method, the duality theorem, and the Kuhn-Tucker theorem.

Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 13300 or MATH 15300 or MATH 16300 or MATH 16310

MATH 19620. Linear Algebra. 100 Units.

This course takes a concrete approach to the basic topics of linear algebra. Topics include vector geometry, systems of linear equations, vector spaces, matrices and determinants, and eigenvalue problems. Prerequisite(s): MATH 13300 or MATH 15200 or MATH 16200. Note(s): Recommended sequence for ECON majors: MATH 19620, STAT 23400, ECON 21000 in consecutive quarters.

Instructor(s): Staff Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 13200 or MATH 15200 or MATH 16200 or MATH 16210.

Note(s): Recommended sequence for ECON majors: MATH 19620, STAT 23400, ECON 21000 in consecutive quarters.

MATH 20000-20100. Mathematical Methods for Physical Sciences I-II.

This sequence is intended for students who are majoring in a department in the Physical Sciences Collegiate Division other than mathematics.

MATH 20000. Mathematical Methods for Physical Sciences I. 100 Units.

MATH 20000 covers multivariable calculus, including the algebra and geometry of Euclidean space, differentiation and integration of functions of several variables, vector valued functions and the classical theorems of vector analysis (i.e., theorems of Green, Gauss, and Stokes), and sequences and series of numbers and functions, including an introduction to Fourier series.

Terms Offered: Autumn Winter

Prerequisite(s): MATH 13300 or 15300 or 16300 or 16310; entering students by invitation only, based on superior performance on the Calculus Accreditation Exam

MATH 20100. Mathematical Methods for Physical Sciences II. 100 Units.

MATH 20100 introduces ordinary differential equations (e.g., first and second order linear differential equations, series solutions, and the Laplace transform) and complex analysis (i.e., basic properties of the complex plane and analytic functions through Cauchy's theorem).

Terms Offered: Spring Winter

Prerequisite(s): MATH 20000 or (MATH 19520 and MATH 19620)

MATH 20100. Mathematical Methods for Physical Sciences II. 100 Units.

MATH 20100 introduces ordinary differential equations (e.g., first and second order linear differential equations, series solutions, and the Laplace transform) and complex analysis (i.e., basic properties of the complex plane and analytic functions through Cauchy's theorem).

Terms Offered: Spring Winter

Prerequisite(s): MATH 20000 or (MATH 19520 and MATH 19620)

MATH 20250. Abstract Linear Algebra. 100 Units.

This is a theoretical course in linear algebra intended for students taking higher level mathematics courses. Topics include vector spaces and linear transformations, matrices and the algebra of matrices, determinants and their properties, the geometry of \mathbb{R}^n and \mathbb{C}^n , bases, coordinates and change of basis, eigenvalues, eigenvectors, characteristic polynomial, diagonalization, special forms including QR factorization and Singular Value Decomposition, and applications.

Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 16300 or MATH 16310 or MATH 15910 or MATH 15900 or MATH 19900

MATH 20300-20400-20500. Analysis in \mathbb{R}^n I-II-III.

This three-course sequence is intended for students who plan to major in mathematics or who require a rigorous treatment of analysis in several dimensions. Both theoretical and problem solving aspects of multivariable calculus are treated carefully. All courses in the sequence require experience with a theoretical treatment of the real numbers, and hence MATH 20300 has a prerequisite of either MATH 16300 or MATH 15910. Additionally, MATH 20400 requires a serious treatment of linear algebra, and thus has a prerequisite of either MATH 20250 or STAT 24300. MATH 20300 covers the construction of the real numbers, the topology of \mathbb{R}^n including the Bolzano-Weierstrass and Heine-Borel theorems, and a detailed treatment of abstract metric spaces, including convergence and completeness, compact sets, continuous mappings, and more. MATH 20400 covers differentiation in \mathbb{R}^n including partial derivatives, gradients, the total derivative, the Chain Rule, optimization problems, vector-valued functions, and the Inverse and Implicit Function Theorems. MATH 20500 covers integration in \mathbb{R}^n including Fubini's Theorem and iterated integration, line and surface integrals, differential forms, and the theorems of Green, Gauss, and Stokes. This sequence is the basis for all advanced courses in analysis and topology.

MATH 20300. Analysis in \mathbb{R}^n I. 100 Units.

MATH 20300 covers the construction of the real numbers, the topology of \mathbb{R}^n including the Bolzano-Weierstrass and Heine-Borel theorems, and a detailed treatment of abstract metric spaces, including convergence and completeness, compact sets, continuous mappings, and more.

Instructor(s): Staff Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 16300 or MATH 16310 or MATH 15910 or MATH 15900 or MATH 19900

MATH 20400. Analysis in \mathbb{R}^n II. 100 Units.

MATH 20400 covers differentiation in \mathbb{R}^n including partial derivatives, gradients, the total derivative, the Chain Rule, optimization problems, vector-valued functions, and the Inverse and Implicit Function Theorems.

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20700 or ((MATH 20300 or MATH 20310) AND (MATH 20250 or STAT 24300))

MATH 20500. Analysis in \mathbb{R}^n III. 100 Units.

MATH 20500 covers integration in \mathbb{R}^n including Fubini's Theorem and iterated integration, line and surface integrals, differential forms, and the theorems of Green, Gauss, and Stokes

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20400 or MATH 20410 or MATH 20800

MATH 20400. Analysis in \mathbb{R}^n II. 100 Units.

MATH 20400 covers differentiation in \mathbb{R}^n including partial derivatives, gradients, the total derivative, the Chain Rule, optimization problems, vector-valued functions, and the Inverse and Implicit Function Theorems.

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20700 or ((MATH 20300 or MATH 20310) AND (MATH 20250 or STAT 24300))

MATH 20500. Analysis in \mathbb{R}^n III. 100 Units.

MATH 20500 covers integration in \mathbb{R}^n including Fubini's Theorem and iterated integration, line and surface integrals, differential forms, and the theorems of Green, Gauss, and Stokes

Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20400 or MATH 20410 or MATH 20800

MATH 20310-20410-20510. Analysis in \mathbb{R}^n I (accelerated); Analysis in \mathbb{R}^n II (accelerated); Analysis in \mathbb{R}^n III (accelerated)

This sequence is an accelerated version of MATH 20300-20400-20500 Analysis in \mathbb{R}^n I-II-III.

MATH 20310. Analysis in \mathbb{R}^n I (accelerated) 100 Units.

This is an accelerated version of MATH 20300.

Instructor(s): Staff Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 16300 or MATH 16310 or MATH 15910 or MATH 15900 or MATH 19900. Students must have received a grade of B+ or better in MATH 16300, 16310, 15900, or 15910 in order to register for the accelerated Analysis sequence.

MATH 20410. Analysis in \mathbb{R}^n II (accelerated) 100 Units.

This is an accelerated version of MATH 20400.

Instructor(s): Staff Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20700 or (MATH 20310 AND (MATH 20250 or STAT 24300))

MATH 20510. Analysis in \mathbb{R}^n III (accelerated) 100 Units.

This is an accelerated version of MATH 20500.

Instructor(s): Staff Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20800 or MATH 20410

MATH 20410. Analysis in \mathbb{R}^n II (accelerated) 100 Units.

This is an accelerated version of MATH 20400.

Instructor(s): Staff Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20700 or (MATH 20310 AND (MATH 20250 or STAT 24300))

MATH 20510. Analysis in \mathbb{R}^n III (accelerated) 100 Units.

This is an accelerated version of MATH 20500.

Instructor(s): Staff Terms Offered: Autumn Spring Winter

Prerequisite(s): MATH 20800 or MATH 20410

MATH 20700-20800-20900. Honors Analysis in \mathbb{R}^n I-II-III.

This highly theoretical sequence in analysis is intended for the most able students. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

MATH 20700. Honors Analysis in \mathbb{R}^n I. 100 Units.

This is the first course in a highly theoretical sequence in analysis, and is intended for the most able students. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): Invitation only, based on performance on the Calculus Accreditation Exam

MATH 20800. Honors Analysis in \mathbb{R}^n II. 100 Units.

This is the second course in a highly theoretical sequence in analysis. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 20700

MATH 20900. Honors Analysis in \mathbb{R}^n III. 100 Units.

This is the third course in a highly theoretical sequence in analysis. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 20800

MATH 20800. Honors Analysis in \mathbb{R}^n II. 100 Units.

This is the second course in a highly theoretical sequence in analysis. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 20700

MATH 20900. Honors Analysis in \mathbb{R}^n III. 100 Units.

This is the third course in a highly theoretical sequence in analysis. Topics include the real number system, metric spaces, basic functional analysis, and the Lebesgue integral.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 20800

MATH 21100. Basic Numerical Analysis. 100 Units.

This course covers direct and iterative methods of solution of linear algebraic equations and eigenvalue problems. Topics include numerical differentiation and quadrature for functions of a single variable, approximation by polynomials and piecewise polynomial functions, approximate solution of ordinary differential equations, and solution of nonlinear equations.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 20000 or 20250 or 20400 or 20410

MATH 21200. Advanced Numerical Analysis. 100 Units.

This course covers topics similar to those of Math 21100 but at a more rigorous level. The emphasis is on proving all of the results. Previous knowledge of numerical analysis is not required. Programming is also not required. The course makes extensive use of the material developed in the analysis sequence (ending in Math 20500 or Math 20900) and provides an introduction to other areas of analysis such as functional analysis and operator theory.

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): MATH 20500 or 20510 or 20900

MATH 23500. Markov Chains, Martingales, and Brownian Motion. 100 Units.

This course discusses three of the most important types of stochastic processes: Markov chains (in both discrete and continuous time), martingales (the mathematical model of "fair games"), and Brownian motion (random continuous motion). Applications will include random walk, queueing theory, and branching processes, and may also include other areas such as optimal stopping or stochastic integration.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): STAT 25100 or STAT 25150, or STAT 24400, or MATH 20500/MATH 20510/MATH 20900 and permission of the instructor

MATH 24100. Topics in Geometry. 100 Units.

This course focuses on the interplay between abstract algebra (group theory, linear algebra, and the like) and geometry. Several of the following topics are covered: affine geometry, projective geometry, bilinear forms, orthogonal geometry, and symplectic geometry.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 25500 or 25800

Note(s): This course is offered in alternate years.

MATH 24200. Algebraic Number Theory. 100 Units.

Topics include factorization in Dedekind domains, integers in a number field, prime factorization, basic properties of ramification, and local degree.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 25500 or 25800

MATH 24300. Intro To Algebraic Curves. 100 Units.

This course covers the projective line and plane curves, both affine and projective. We also study conics and cubics, as well as the group law on the cubic. Abstract curves associated to function fields of one variable are discussed, along with the genus of a curve and the Riemann-Roch theorem. Curves of low genus are emphasized. Although the formal prerequisite is MATH 25500 or 25800, MATH 25600 or 25900 is strongly recommended.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 25500 or 25800, or consent of instructor

Note(s): This course is offered in alternate years.

MATH 24400. Introduction to Algebraic Geometry. 100 Units.

This is a first course in algebraic geometry. Topics include: affine and projective varieties; coordinate rings; the Zariski topology; Nullstellensatz; Hilbert basis Theorem; the dictionary between algebraic geometry and commutative algebra; rational functions and morphisms; smoothness; theory of dimension. Other possible topics might include: the classification of plane cubics; elliptic curves; 27 lines on a cubic surface; introduction to the theory of curves (degree, divisors, Bezout's Theorem, etc.). Although the formal algebra prerequisite is MATH 25500 or MATH 25800, in fact MATH 25600 or MATH 25900 is strongly recommended. Additionally, MATH 27000 and MATH 26200 are strongly recommended. Prerequisite(s): (MATH 20500 or MATH 20900) and (MATH 25500 or MATH 25800) Note(s): This course is offered in alternate years.

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): (MATH 20500 or MATH 20510 or MATH 20900) and (MATH 25500 or MATH 25800)

Note(s): This course is offered in alternate years.

MATH 25400-25500. Basic Algebra I-II.

This is the sequence in basic algebra. It requires a prior serious treatment of linear algebra and thus has a prerequisite of MATH 20250. MATH 25400 covers groups, subgroups, permutation groups, group actions, and Sylow Theorems. MATH 25500 covers rings and ideals, PIDs, Euclidean domains, UFDs, fields and field extensions, and the fundamentals of Galois theory.

MATH 25400. Basic Algebra-I. 100 Units.

This is the sequence in basic algebra. It requires a prior serious treatment of linear algebra, and thus has a prerequisite of MATH 20250. MATH 25400 covers groups, subgroups, permutation groups, and group actions. MATH 25500 covers rings and ideals, PIDs, Euclidean domains, UFDs, fields and field extensions, and canonical forms of matrices, quadratic forms, and multilinear algebra. MATH 25600 covers the Sylow theorems and the fundamentals of Galois theory. This course covers groups, subgroups, permutation groups, and group actions. Prerequisite(s): MATH 20250 or MATH 20700

Terms Offered: Autumn Winter

Prerequisite(s): MATH 20250 or MATH 20700

MATH 25500. Basic Algebra II. 100 Units.

This course covers rings and ideals, PIDs, Euclidean domains, UFDs, fields and field extensions, modules and canonical forms of matrices, quadratic forms, and multilinear algebra.

Terms Offered: Spring Winter

Prerequisite(s): MATH 25400 or MATH 25700

MATH 25500. Basic Algebra II. 100 Units.

This course covers rings and ideals, PIDs, Euclidean domains, UFDs, fields and field extensions, modules and canonical forms of matrices, quadratic forms, and multilinear algebra.

Terms Offered: Spring Winter

Prerequisite(s): MATH 25400 or MATH 25700

MATH 25700-25800-25900. Honors Basic Algebra I-II-III.

This sequence is an accelerated version of MATH 25400-25500-25600 that is open only to students who have achieved a B- or better in prior mathematics courses. Topics include the theory of finite groups, commutative and noncommutative ring theory, modules, linear and multilinear algebra, and quadratic forms. We also cover basic field theory, the structure of p-adic fields, and Galois theory.

MATH 25700. Honors Basic Algebra I. 100 Units.

Topics in MATH 25700 include the theory of finite groups, up through and including the proofs of the Sylow Theorems.

Terms Offered: Autumn

Prerequisite(s): MATH 20700 or MATH 20250; no entering student may begin this sequence in their first term.

MATH 25800. Honors Basic Algebra II. 100 Units.

Topics in MATH 25800 include commutative and noncommutative ring theory, modules, and field extensions.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 25700

MATH 25900. Honors Basic Algebra III. 100 Units.

Topics in this course include basic field theory, the structure of p-adic fields, and Galois theory.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 25800

MATH 25800. Honors Basic Algebra II. 100 Units.

Topics in MATH 25800 include commutative and noncommutative ring theory, modules, and field extensions.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 25700

MATH 25900. Honors Basic Algebra III. 100 Units.

Topics in this course include basic field theory, the structure of p-adic fields, and Galois theory.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 25800

MATH 26200. Point-Set Topology. 100 Units.

This course examines topology on the real line, topological spaces, connected spaces and compact spaces, identification spaces and cell complexes, and projective and other spaces. With MATH 27400, it forms a foundation for all advanced courses in analysis, geometry, and topology.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 20300 or 20310 or 20700, and 25400 or 25700

MATH 26300. Introduction to Algebraic Topology. 100 Units.

Topics include the fundamental group of a space; Van Kampen's theorem; covering spaces and groups of covering transformation; existence of universal covering spaces built up out of cells; and theorems of Gauss, Brouwer, and Borsuk-Ulam.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 26200

MATH 26700. Introduction to Representation Theory of Finite Groups. 100 Units.

Topics include group algebras and modules, semisimple algebras and the theorem of Maschke; characters, character tables, orthogonality relations and calculation; and induced representations and characters. Applications to permutation groups and solvability of groups are also included

Instructor(s): Staff Terms Offered: Autumn

Prerequisite(s): MATH 25800 or 25500

MATH 26800. Introduction to Commutative Algebra. 100 Units.

Topics include basic definitions and properties of commutative rings and modules, Noetherian and Artinian modules, exact sequences, Hilbert basis theorem, tensor products, localizations of rings and modules, associated primes and primary decomposition, Artin-Rees Lemma, Krull intersection theorem, completions, dimension theory of Noetherian rings, integral extensions, normal domains, Dedekind domains, going up and going down theorems, dimension of finitely generated algebras over a field, Affine varieties, Hilbert Nullstellensatz, dimension of affine varieties, product of affine varieties, and the dimension of intersection of subvarieties.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 25800 or 25500

MATH 27000. Basic Complex Variables. 100 Units.

Topics include complex numbers, elementary functions of a complex variable, complex integration, power series, residues, and conformal mapping.

Instructor(s): Staff Terms Offered: Autumn, Spring, Winter

Prerequisite(s): MATH 20500 or 20510 or 20900

MATH 27100. Measure and Integration. 100 Units.

Construction of Lebesgue measure, measurable sets and functions. The Lebesgue integral, convergence theorems.

Relationship between Lebesgue and Riemann integral. The L^p spaces, completeness and duality. Other related topics time permitting.

Instructor(s): Staff Terms Offered: Winter

Prerequisite(s): MATH 20500 or MATH 20510

MATH 27200. Basic Functional Analysis. 100 Units.

Review of metric spaces, normed spaces and inner product spaces. Banach spaces and Hilbert spaces. Bounded linear operators. The 3 basic principles of functional analysis: the closed graph theorem, the open mapping theorem and the uniform boundedness principle. Duality and weak topologies. Weak and weak star compactness. Hilbert spaces, orthogonal expansions and spectral theorem. Applications of these concepts.

Terms Offered: Spring

Prerequisite(s): MATH 27000 and (MATH 20900 or MATH 27100)

MATH 27300. Basic Theory of Ordinary Differential Equations. 100 Units.

This course is an introduction to the theory of ordinary differential equations in Euclidean space. Topics covered include: first-order equations of one variable, solving higher order systems via reduction of order, linear ODEs in arbitrary dimension, real Jordan form and the matrix exponential, variation of parameters, existence and uniqueness of solutions for Lipschitz vector fields, local analysis near equilibria, stability of solutions, introduction to dynamical systems and the global analysis of flows.

Instructor(s): Staff Terms Offered: Autumn, Winter

Prerequisite(s): MATH 20500 or MATH 20510 or MATH 20900 or PHYS 22100

MATH 27400. Introduction to Differentiable Manifolds and Integration on Manifolds. 100 Units.

Topics include exterior algebra; differentiable manifolds and their basic properties; differential forms; integration on manifolds; and the theorems of Stokes, DeRham, and Sard. With MATH 26200, this course forms a foundation for all advanced courses in analysis, geometry, and topology.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 26200

MATH 27500. Basic Theory of Partial Differential Equations. 100 Units.

This course covers classification of second-order equations in two variables, wave motion and Fourier series, heat flow and Fourier integral, Laplace's equation and complex variables, second-order equations in more than two variables, Laplace operators, spherical harmonics, and associated special functions of mathematical physics.

Instructor(s): Staff Terms Offered: Spring

Prerequisite(s): MATH 27000 and MATH 27300

MATH 27700-27800. Mathematical Logic I-II.

Mathematical Logic I-II

MATH 27700. Mathematical Logic-I. 100 Units.

This course introduces mathematical logic. Topics include propositional and predicate logic and the syntactic notion of proof versus the semantic notion of truth (e.g., soundness, completeness). We also discuss the Gödel completeness theorem, the compactness theorem, and applications of compactness to algebraic problems. Prerequisite(s): MATH 25400 or MATH 25700 or (CMSC 15400 and (MATH 15910 or MATH 15900 or MATH 19900 or MATH 16300))

Equivalent Course(s): CMSC 27700

Terms Offered: Autumn

Prerequisite(s): MATH 25400 or 25700; open to students who are majoring in computer science who have taken CMSC 15400 along with MATH 16300 or MATH 15900 or MATH 19900

Equivalent Course(s): CMSC 27700

MATH 27800. Mathematical Logic II. 100 Units.

Topics include number theory, Peano arithmetic, Turing compatibility, unsolvable problems, Gödel's incompleteness theorem, undecidable theories (e.g., the theory of groups), quantifier elimination, and decidable theories (e.g., the theory of algebraically closed fields).

Terms Offered: Winter

Prerequisite(s): MATH 27700 or equivalent

Equivalent Course(s): CMSC 27800

MATH 27800. Mathematical Logic II. 100 Units.

Topics include number theory, Peano arithmetic, Turing compatibility, unsolvable problems, Gödel's incompleteness theorem, undecidable theories (e.g., the theory of groups), quantifier elimination, and decidable theories (e.g., the theory of algebraically closed fields).

Terms Offered: Winter

Prerequisite(s): MATH 27700 or equivalent

Equivalent Course(s): CMSC 27800

MATH 28000. Introduction to Formal Languages. 100 Units.

This course is a basic introduction to computability theory and formal languages. Topics include automata theory, regular languages, context-free languages, and Turing machines.

Instructor(s): S. Kurtz Terms Offered: Spring

Prerequisite(s): CMSC 12300 or CMSC 15400, or MATH 15900 or MATH 25500.

Equivalent Course(s): CMSC 28000

MATH 28100. Introduction to Complexity Theory. 100 Units.

Computability topics are discussed (e.g., the s-m-n theorem and the recursion theorem, resource-bounded computation).

This course introduces complexity theory. Relationships between space and time, determinism and non-determinism, NP-completeness, and the P versus NP question are investigated.

Instructor(s): K. Mulmuley Terms Offered: Autumn

Prerequisite(s): CMSC 27100, or MATH 15900 or MATH 25500; experience with mathematical proofs.

Equivalent Course(s): CMSC 28100

MATH 28410. Honors Combinatorics. 100 Units.

Methods of enumeration, construction, and proof of existence of discrete structures are discussed in conjunction with the basic concepts of probability theory over a finite sample space. Enumeration techniques are applied to the calculation of probabilities, and, conversely, probabilistic arguments are used in the analysis of combinatorial structures. Other topics include basic counting, linear recurrences, generating functions, Latin squares, finite projective planes, graph theory, Ramsey theory, coloring graphs and set systems, random variables, independence, expected value, standard deviation, and Chebyshev's and Chernoff's inequalities.

Instructor(s): L. Babai Terms Offered: Spring

Prerequisite(s): MATH 15900 or MATH 25400, or CMSC 27100, or by consent. Experience with mathematical proofs.

Note(s): This course is offered in alternate years.

Equivalent Course(s): CMSC 27410

MATH 29520. Introduction to Error-Correcting Codes. 100 Units.

Cyclic codes, BCH codes, Golay codes, Shannon's Theorem, and codes approaching Shannon's bounds will be covered.

Applications to electrical engineering, combinatorics, and group theory will be discussed.

Instructor(s): Staff Terms Offered: Winter, in alternate years

Prerequisite(s): MATH 25500 or 25800

MATH 29700. Proseminar in Mathematics. 100 Units.

Consent of instructor and departmental counselor. Students are required to submit the College Reading and Research Course Form. Must be taken for a quality grade.

Instructor(s): Staff Terms Offered: Autumn, Spring, Winter

Prerequisite(s): Completion of general education mathematics sequence



Font Notice

This document should contain certain fonts with restrictive licenses. For this draft, substitutions were made using less legally restrictive fonts. Specifically:

Times was used instead of Trajan.

Times was used instead of Palatino.

The editor may contact Leepfrog for a draft with the correct fonts in place.